

Gravity and Mirror Gravity in Plebanski Formulation

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We present several theories of four-dimensional gravity in the Plebanski formulation, in which the tetrads and the connections are the independent dynamical variables. We consider the relation between different versions of gravitational theories: Einstenian, dual, 'mirror' gravities and gravity with torsion. According to Plebanski's assumption, our world, in which we live, is described by the self-dual left-handed gravity. We propose that if the Mirror World exists in Nature, then the 'mirror gravity' is the right-handed anti-self-dual gravity with broken mirror parity. Considering a special version of the Riemann–Cartan space-time, which has torsion as additional geometric property, we have shown that in the Plebanski formulation the ordinary and dual sectors of gravity, as well as the gravity with torsion, are equivalent. In this context, we have also developed a 'pure connection gravity' – a diffeomorphism-invariant gauge theory of gravity. We have calculated the partition function and the effective Lagrangian of this four-dimensional gravity and have investigated the limit of this theory at small distances.

I. INTRODUCTION. PLEBANSKI'S FORMULATION OF GRAVITY

The main idea of Plebanski's formulation of the 4-dimensional theory of gravity [1] is the construction of the gravitational action from the product of two 2-forms [1–7]. These 2-forms are constructed using the connection A^{IJ} and tetrads θ^I as independent dynamical variables.

We consider a Lorentzian metric. The signature of the metric tensor denoted by the pair of integers (p, q) is chosen as the Lorentzian signature $(1, 3)$.

The tetrads θ^I are used instead of the metric $g_{\mu\nu}$. Both A^{IJ} and θ^I are 1-forms:

$$A^{IJ} = A_{\mu}^{IJ} dx^{\mu} \quad \text{and} \quad \theta^I = \theta_{\mu}^I dx^{\mu}. \quad (1)$$

Here the indices $I, J = 0, 1, 2, 3$ refer to the space-time with Minkowski metric η_{IJ} : $\eta^{IJ} = \text{diag}(1, -1, -1, -1)$. This is a flat space which is tangential to the curved space with the metric $g_{\mu\nu}$. The world interval is represented as $ds^2 = \eta_{IJ} \theta^I \otimes \theta^J$, i.e.

$$g_{\mu\nu} = \eta_{IJ} \theta_{\mu}^I \otimes \theta_{\nu}^J. \quad (2)$$

Considering the case of the Minkowski flat space-time with the group of symmetry $SO(1, 3)$, we have the capital latin indices $I, J, \dots = 0, 1, 2, 3$, which are vector indices under the rotation group $SO(1, 3)$.

In the well-known Plebanski's BF-theory of general relativity (GR) [1], the gravitational action (with zero cosmological constant) is

$$\int \epsilon^{IJKL} B^{IJ} \wedge F^{KL}, \quad (3)$$

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where B^{IJ} and F^{IJ} are the following 2-forms:

$$B^{IJ} = \theta^I \wedge \theta^J = \frac{1}{2} \theta_\mu^I \theta_\nu^J dx^\mu \wedge dx^\nu, \quad (4)$$

and

$$F^{IJ} = \frac{1}{2} F_{\mu\nu}^{IJ} dx^\mu \wedge dx^\nu. \quad (5)$$

Here the tensor $F_{\mu\nu}^{IJ}$ is the field strength of the spin connection A_μ^{IJ} :

$$F_{\mu\nu}^{IJ} = \partial_\mu A_\nu^{IJ} - \partial_\nu A_\mu^{IJ} - [A_\mu, A_\nu]^{IJ}, \quad (6)$$

which determines the Riemann–Cartan curvature:

$$R_{\mu\nu\lambda\kappa} = F_{\mu\nu}^{IJ} \theta_\lambda^I \theta_\kappa^J. \quad (7)$$

Now the question is how many different products of two simple 2-forms can be constructed in the space-time of the 4-dimensional GR. Then all of these 4-forms can be considered as terms of the integrand of the gravitational action.

We have only a few possibilities for such 4-forms:

$$B^{IJ} \wedge B^{IJ}, \quad B^{IJ} \wedge F^{IJ}, \quad F^{IJ} \wedge F^{IJ}, \quad (8)$$

their dual counterparts:

$$\epsilon^{IJKL} B^{IJ} \wedge B^{KL}, \quad \epsilon^{IJKL} B^{IJ} \wedge F^{KL}, \quad \epsilon^{IJKL} F^{IJ} \wedge F^{KL}, \quad (9)$$

and

$$S^I \wedge S^I. \quad (10)$$

The 2-form

$$S^I = \frac{1}{2} S_{\mu\nu}^I dx^\mu \wedge dx^\nu \quad (11)$$

contains the torsion $S_{\mu\nu}^I$ [8]:

$$S_{\mu\nu}^I = D_\mu^{IJ} \theta_\nu^J - D_\nu^{IJ} \theta_\mu^J, \quad (12)$$

where

$$D_\mu^{IJ} = \delta^{IJ} \partial_\mu - A_\mu^{IJ} \quad (13)$$

is the covariant derivative.

In the Plebanski BF-theory, the gravitational action with nonzero cosmological constant Λ is presented by the integral:

$$I_{GR} = \frac{1}{\kappa^2} \int \epsilon^{IJKL} \left(B^{IJ} \wedge F^{KL} + \frac{\Lambda}{4} B^{IJ} \wedge B^{KL} \right), \quad (14)$$

where $\kappa^2 = 8\pi G$, G is the gravitational constant, and the reduced Planck mass is $M_{Pl}^{red.} = 1/\sqrt{8\pi G}$.

Below we use units $\kappa = 1$ (in these units $M_{Pl}^{red.} = 1$).

The "dual sector" of gravity is described by the following integral [9]:

$$I_{dual\ GR} = 2 \int \left(B^{IJ} \wedge F^{IJ} + \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ} \right) = \int \epsilon^{IJKL} \left(B^{IJ} \wedge F^{*KL} + \frac{\Lambda}{4} B^{IJ} \wedge B^{*KL} \right), \quad (15)$$

where $F^{*IJ} = \frac{1}{2} \epsilon^{IJKL} F^{KL}$ is the dual tensor.

The paper is organized as follows. In Section I we review the main idea of Plebanski to construct the 4-dimensional theory of gravity considering the integrand of the gravitational action as the product of two 2-forms

containing only tetrads and connections which are independent dynamical variables. The BF-theories are presented for the ordinary and dual gravity in the Minkowski flat space-time. In Section II we construct the self-dual left-handed and the anti-self-dual right-handed gravitational worlds. In Section III we investigate the 'mirror gravity' existing in the 'Mirror World', which describes the states of particle physics with opposite chirality. We assume that the mirror gravity which has to interact with the states of opposite "right-handed" chirality can be described by the anti-self-dual right-handed action of gravity. Using modern astrophysical and cosmological measurements, we consider the broken mirror parity (MP), and discuss the communications between visible and hidden worlds. Section IV is devoted to gravity with torsion. It was shown that in the self-dual Plebanski formulation of GR, gravity with torsion coincides with the ordinary and dual versions of gravity. A new type of gauge transformation in Riemann–Cartan space-time is considered: Einstein's theory of gravity described only by curvature can be rewritten as Einstein's teleparallel theory of gravity described only by torsion. In Section V the equations of motion resulting from Plebanski's gravitational action are used to construct the action containing only the connection and auxiliary fields. Einstein's equations are investigated in terms of Plebanski's theory of gravity. Section VI is devoted to the diffeomorphism invariant gauge theory of gravity, which is a new type of gravitational theory with a 'pure connection' formulation of GR. The calculation of the partition function and the effective Lagrangian of this 4-dimensional gravity is presented, as well as field equations. The limits of this theory at small and large distances are investigated. In the asymptotic limit of this theory, we have gravity in the flat (Euclidean or Minkowski) space-time and the effective gravitational coupling constant is given by the bare cosmological constant Λ : $g_{eff} = \Lambda/2$. At large distances we predict a more complicated theory of gravity. The perspectives of the quantum theory of Plebanski's gravity are discussed in Subsections VIA and VIB. Section VII contains a summary and conclusions.

II. THE "LEFT-HANDED" AND "RIGHT-HANDED" GRAVITY

The next step of the study of the Plebanski BF-theory is the consideration of the decomposition of $SO(1,3)$ -group of GR on left- and right-handed sectors.

For any antisymmetric tensor A^{IJ} there exists a dual tensor given by the Hodge star dual operation on the indexes I, J of flat space:

$$A^{*IJ} = \frac{1}{2}\epsilon^{IJKL}A^{KL}. \quad (16)$$

This antisymmetric tensor A^{IJ} can be split into a self-dual component A^+ and an anti-self-dual component A^- , according to the relation:

$$A^\pm = \frac{1}{2}(A \pm A^*). \quad (17)$$

As with any Lie group, the best way to study many aspects of the Lorentz group is via its Lie algebra. Since the Lorentz group is $SO(1,3)$, its Lie algebra is reducible and can be decomposed into two copies of the Lie algebra of $SL(2, R)$:

$$SO(1,3) = SL(2, R)_{left} \times SL(2, R)_{right}. \quad (18)$$

As it was shown explicitly in [10], this is the Minkowski space analog of the $SO(4) = SU(2)_{left} \times SU(2)_{right}$ decomposition in a Euclidean space.

The complex Lorentz algebra splits into two complex $SO(3)$ algebra called the self-dual (left-handed) and anti-self-dual (right-handed) components [1, 2]:

$$SO(1,3, C) = SO(3, C)_{left} \times SO(3, C)_{right}. \quad (19)$$

Because of this split, the curvature of the self-dual components of the connection is the self-dual component of the curvature.

In particle physics, a state that is invariant under one of these copies of $SO(3, C)$ is said to have chirality, and is either left-handed or right-handed, according to which copy of $SO(3, C)$ it is invariant under.

Self-dual tensors transform non-trivially only under $SO(3, C)_{left}$ and are invariant under $SO(3, C)_{right}$. By this reason, they are called "left-handed" tensors. Similarly, anti-self-dual tensors, non-trivially transforming only under $SO(3, C)_{right}$, are called "right-handed" tensors. These self-dual and anti-self-dual tensors $A^{\pm IJ}$ have only three independent components given by $IJ = 0i$, $i = 1, 2, 3$:

$$A^{\pm i} = \pm 2A^{\pm 0i}, \quad (20)$$

which transform as adjoint vector components under the corresponding $SU(2)$ -group.

Such a decomposition shows (see Refs. [2–5]) that the actions I_{GR} and $I_{dual\ GR}$, given by Eqs. (14) and (15), respectively, can be represented in terms of the "left-handed" and "right-handed" gravity:

$$I_{GR} = \int [\Sigma^i \wedge F^i - \bar{\Sigma}^i \wedge \bar{F}^i + \Lambda(\Sigma^i \wedge \Sigma^i - \bar{\Sigma}^i \wedge \bar{\Sigma}^i)], \quad (21)$$

and

$$I_{dual\ GR} = \int [\Sigma^i \wedge F^i + \bar{\Sigma}^i \wedge \bar{F}^i + \Lambda(\Sigma^i \wedge \Sigma^i + \bar{\Sigma}^i \wedge \bar{\Sigma}^i)], \quad (22)$$

where

$$F_{\mu\nu}^{\pm i} = \partial_\mu A_\nu^{\pm i} - \partial_\nu A_\mu^{\pm i} + \epsilon^{ijk} A_\mu^{\pm j} A_\nu^{\pm k}, \quad (23)$$

$F \equiv F^+$, $\bar{F} \equiv F^-$, $\Sigma \equiv \Sigma^+$, $\bar{\Sigma} \equiv \Sigma^-$, and the left-handed and right-handed $\Sigma^{\pm i}$ are given by:

$$\Sigma^+ = i\theta^0 \wedge \theta^i - \frac{1}{2}\epsilon^{ijk}\theta^j \wedge \theta^k, \quad \Sigma^- = i\theta^0 \wedge \theta^i + \frac{1}{2}\epsilon^{ijk}\theta^j \wedge \theta^k. \quad (24)$$

The choice of the second Killing form in the cosmological term is made so that the full Plebanski action, obtained by adding the so-called simplicity constraints to (21), is equivalent (in the appropriate sector) to ordinary gravity (see also [6]). The so-called simplicity constraints are introduced in the gravitational actions by means of the Lagrange multipliers ψ_{ij} , which are considered in theory as auxiliary fields, symmetric and traceless. Finally, the resulting actions of the Plebanski gravity are [1–7]:

$$I_{GR} = \int [\Sigma^i \wedge F^i - \bar{\Sigma}^i \wedge \bar{F}^i + (\Psi^{-1})_{ij}(\Sigma^i \wedge \Sigma^j - \bar{\Sigma}^i \wedge \bar{\Sigma}^j)], \quad (25)$$

and

$$I_{dual\ GR} = \int [\Sigma^i \wedge F^i + \bar{\Sigma}^i \wedge \bar{F}^i + (\Psi^{-1})_{ij}(\Sigma^i \wedge \Sigma^j + \bar{\Sigma}^i \wedge \bar{\Sigma}^j)], \quad (26)$$

where

$$(\Psi^{-1})_{ij} = \Lambda\delta_{ij} + \psi_{ij}. \quad (27)$$

The stationarity with respect to ψ_{ij} provides the correct number of constraints, reducing the 36 degrees of freedom of (Σ^+, Σ^-) to the 16 degrees of freedom of tetrads θ_μ^I .

Now we can distinguish the two worlds – two sectors of gravity: left-handed gravity and right-handed gravity. The self-dual left-handed gravitational world can be described by the action:

$$I_{(left\ GR)}(\Sigma, A) = \int [\Sigma^i \wedge F^i + (\Psi^{-1})_{ij}\Sigma^i \wedge \Sigma^j], \quad (28)$$

while the anti-self-dual right-handed gravitational world is given by the following action:

$$I_{(right\ GR)}(\bar{\Sigma}, \bar{A}) = \int [\bar{\Sigma}^i \wedge \bar{F}^i + (\Psi^{-1})_{ij}\bar{\Sigma}^i \wedge \bar{\Sigma}^j]. \quad (29)$$

If the anti-self-dual right-handed gravitational world is absent in Nature ($\bar{F} = 0$ and $\bar{\Sigma} = 0$), then gravity is presented only by the self-dual left-handed Plebanski action (28). The main assumption of Plebanski was that our world, in which we live, is a self-dual left-handed gravitational world described by the action (28). The same self-dual formulation of general relativity (GR) was developed later by Ashtekar [11].

If there exist in Nature two parallel worlds with opposite chiralities – Ordinary and Mirror (see below, Section III) – then we must consider the left-handed gravity in the Ordinary world and the right-handed gravity in the Mirror world.

It is not difficult to show [1] that the Plebanski action (28) corresponds to the Einstein–Hilbert action of gravity. In the Minkowski space background, the Einstein–Hilbert action is:

$$I_{EH} = \frac{1}{\kappa^2} \int \left(\frac{1}{2}R - \Lambda_0 \right) \sqrt{-g} d^4x, \quad (30)$$

where R is a scalar curvature, and Λ_0 is Einstein's cosmological constant. Here we have (see Subsection V A):

$$\Lambda_0 = 6\Lambda. \quad (31)$$

According to Eqs. (25) and (26), in the Plebanski self-dual formulation of gravity we have the following equality:

$$I_{GR} = I_{dual\ GR}. \quad (32)$$

Both actions are given by the formula (28).

III. MIRROR WORLD AND MIRROR GRAVITY

Previously, in Refs. [12, 13] we have assumed that there exists in Nature a Mirror World (MW) [14, 15], which is a duplication of our Ordinary World (OW), or shadow Hidden World (HW) [16, 17], parallel to our Ordinary World (OW). This MW (or HW) can explain the origin of dark matter and dark energy.

Postulating the existence of the Mirror World, we confront ourselves with a question: should the mirror gravity be the anti-self-dual right-handed gravity? We assume that we have this case, and the mirror gravitational action is given by Eq. (29) describing the anti-self-dual right-handed gravity.

The MW is a mirror copy of the OW and contains the same particles and types of interactions as our visible world. Lee and Yang were the first [14] to suggest such a duplication of the worlds, which restores the left-right symmetry of Nature. The term 'Mirror Matter' was introduced by Kobzarev, Okun and Pomeranchuk [15]. They suggested the 'Mirror World' as the hidden sector of our Universe, which interacts with the ordinary (visible) world only via gravity or another very weak interaction. This assumption was considered in many papers.

Considering only pure gravity, we can formulate mirror parity (MP) investigating the invariance of the Plebanski gravitational action under the dual symmetry, i.e. under the interchanges:

$$A \leftrightarrow \bar{A} \quad (F \leftrightarrow \bar{F}), \quad \Sigma \leftrightarrow \bar{\Sigma}. \quad (33)$$

Introducing projectors on the spaces of the so-called self- and anti-self-dual tensors, we obtain:

$$P^\pm = \frac{1}{2} \left(\mathcal{I}^{IJKL} \pm \frac{1}{2} \epsilon^{IJKL} \right), \quad (34)$$

where

$$\mathcal{I}^{IJKL} = \frac{1}{2} (\delta_{IK} \delta_{JL} - \delta_{IL} \delta_{JK}). \quad (35)$$

If we represent the gravitational action (14) as

$$I = \int \epsilon^{IJKL} \mathcal{L}^{IJKL}, \quad (36)$$

we can consider the relation:

$$P^+ \mathcal{L}^{IJKL} = \alpha P^- \mathcal{L}^{IJKL}. \quad (37)$$

The dual symmetry gives $\alpha = 1$. In this case the action

$$I(\Sigma, A) = \int [\Sigma^i \wedge F^i + (\Psi^{-1})_{ij} \Sigma^i \wedge \Sigma^j] \quad (38)$$

is invariant under the interchanges (33). Considering the left-handed gravitational action (28) in the OW and the right-handed gravitational action (29) in the MW, we have an unbroken mirror parity (MP). In this case, the bare cosmological constants in the OW and MW are identical:

$$\Lambda_0 = \Lambda_0^{(O)} = \Lambda_0^{(M)}. \quad (39)$$

Let us consider now the Universe with matter fields. Assuming the existence of the mirror world, we can enlarge the Standard Model (SM) gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ to $G_{SM} \times G'_{SM}$, where the gauge group $G'_{SM} = SU(3)'_c \times SU(2)'_R \times U(1)'_Y$ (see Refs. [18, 19] and review [20]) is a mirror of G_{SM} with identical gauge

couplings, under which the matter contents switch their chiralities. Hence, the mirror parity is restored in the Universe where the visible and mirror worlds coexist in the same space-time.

Astrophysical and cosmological observations [21] have revealed the existence of dark matter (DM) which constitutes about 23% of the total energy density of the present Universe. This is five times larger than all the visible matter, $\Omega_{DM} : \Omega_M \simeq 5 : 1$. In parallel to the visible world, the mirror world conserves mirror baryon number and thus protects the stability of the lightest mirror nucleon. Mirror particles have therefore been suggested as candidates for the inferred dark matter in the Universe [22] (see also Refs. [18–20, 23, 24]. This explains the right amount of dark matter, which is generated via the mirror leptogenesis [13, 25], just like the visible matter is generated via ordinary leptogenesis [26].

If the ordinary and mirror worlds are identical, then O- and M-particles should have the same cosmological densities. But this is in conflict with recent astrophysical measurements [21]. Mirror parity (MP) is not conserved, and the ordinary and mirror worlds are not identical [12, 13, 18–20] in the sense that, although the chain of breakings of the gauge groups is the same in both worlds, the energy scales at which these breakings take place are different.

Superstring theory also predicts that there may exist in the Universe another form of matter – hidden ‘shadow matter’, which only interacts with ordinary matter via gravity or gravitational-strength interactions [17]. According to the superstring theory, the two worlds, ordinary and shadow, can be viewed as parallel branes in a higher dimensional space, where O-particles are localized on one brane and hidden particles – on another brane, and gravity propagates in the bulk.

In the presence of matter, the Einstein field equations:

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi GT^{\mu\nu} - \Lambda_0 g^{\mu\nu} \quad (40)$$

contain the energy momentum tensor of matter $T^{\mu\nu}$, and all quantum fluctuations of the matter contribute to the vacuum energy ρ_{vac} of the Universe. The resulting cosmological constant is the effective cosmological constant which is equal to

$$\Lambda_{eff} = \Lambda_0 + 8\pi G\rho_{vac}. \quad (41)$$

If the mirror parity is broken also in the gravitational sector ($\alpha \neq 1$), then we can distinguish bare cosmological constants of the O- and M-worlds:

$$\Lambda_0^{(O)} \neq \Lambda_0^{(M)}. \quad (42)$$

In the units $\kappa = \kappa' = 1$, we have:

$$\Lambda_{eff}^{(O,M)} = \Lambda_0^{(O,M)} + \rho_{vac}^{(O,M)} = \rho_{vac,eff}^{(O,M)}. \quad (43)$$

The vacuum energy densities $\rho_{vac}^{(O,M)}$ are given by a trace of the stress-energy tensor of matter $T_{\mu\nu}^{(O,M)}$ in the O- and M-worlds. The effective vacuum energy of the Universe is the sum:

$$\Lambda_{eff} = \Lambda_{eff}^{(O)} + \Lambda_{eff}^{(M)} = \rho_{vac,eff}. \quad (44)$$

Since the ordinary and mirror worlds are not identical, we have:

$$\Lambda_{eff}^{(O)} \neq \Lambda_{eff}^{(M)}. \quad (45)$$

With the aim to explain the tiny value of the dark energy density $\rho_{DE} = \Lambda_{eff} = \rho_{vac,eff} \simeq (2.3 \times 10^{-3} \text{ eV})^4$, verified by astronomical and cosmological observations [21], we have several possibilities. For example:

I) The Universe is described by the theory $G_{SM} \times G'_{SM}$ with broken mirror parity [18–20]. We can assume that

$$\Lambda_{eff}^{(O)} = \Lambda_0^{(O)} + \rho_{vac}^{(SM)} \simeq 0, \quad \Lambda_{eff}^{(M)} = \Lambda_0^{(M)} + \rho_{vac}^{(SM')} \simeq \rho_{DE}.$$

If the supersymmetry breaking scale is the same in O- and M-worlds, then $\rho_{vac}^{(SM)} \simeq \rho_{vac}^{(SM')}$ and

$$\rho_{DE} \simeq \Lambda_0^{(M)} - \Lambda_0^{(O)}.$$

Then the condensate of gravitational fields can explain the value of ρ_{DE} .

II) In Refs. [12, 13] we considered the theory of the E_6 unification with different types of breaking in the visible (O) and hidden (M) worlds. We assumed that at the first stage of the Universe we had unbroken mirror parity: $\Lambda_0^{(O)} = \Lambda_0^{(M)}$ and $E_6 = E'_6$. Finally, E_6 was broken to G_{SM} , and E'_6 underwent the breaking to $G'_{SM} \times SU(2)'_\theta$, where $SU(2)'_\theta$ is the group whose gauge fields are massless vector particles, 'thetons' [27]. These 'thetons' have a macroscopic confinement radius $1/\Lambda'_\theta$. The estimate given by Refs. [12, 13] confirms the scale $\Lambda'_\theta \sim 10^{-3}$ eV.

We assumed that $\Lambda_{eff}^{(O)} = \Lambda_0^{(O)} + \rho_{vac}^{(SM)} \simeq 0$, and $\Lambda_{eff}^{(M)} = \Lambda_0^{(M)} + \rho_{vac}^{(SM')} + \rho_{vac}^{(\theta')} \neq 0$. Assuming the same supersymmetry breaking scale in O- and M-worlds, we considered: $\Lambda_0^{(M)} + \rho_{vac}^{(SM')} \simeq 0$. Then:

$$\Lambda_{eff} = \Lambda_{eff}^{(M)} = \rho_{DE} \simeq \rho_{vac}^{(\theta')} \simeq (\Lambda'_\theta)^4 \simeq (2.3 \times 10^{-3} \text{ eV})^4, \quad (46)$$

in accordance with cosmological measurements [21].

A. Communications between Visible and Hidden Worlds

Mirror particles have not been seen so far, because the communication between visible and hidden worlds is hard.

There are several fundamental ways by which the hidden world can communicate with our visible world.

I) It is necessary to assume that the self-dual gravity interacts not only with visible matter, but also with mirror matter, and anti-self-dual gravity also interacts with matter and mirror matter (see Subsection VI A). It is then to be expected that a fraction of the mirror matter exists in the form of mirror galaxies, mirror stars, mirror planets etc. These objects can be detected using gravitational microlensing [28].

II) There exists the kinetic mixing between the electromagnetic field strength tensors for visible and mirror photons:

$$L_\gamma^{mix} = -\frac{\epsilon\gamma}{2} F_{\gamma\mu\nu} F'^{\mu\nu}_\gamma. \quad (47)$$

The photon–mirror photon mixing induces oscillations between orthopositronium and mirror orthopositronium [29]. Orthopositronium could then turn into mirror orthopositronium and then decay into mirror photons.

Besides these interactions, the hidden world can communicate with our visible world by mass mixings between visible and mirror neutrinos [30] (neutrino–mirror neutrino oscillations). Also mirror neutrons can oscillate to ordinary neutrons [31]. It is expected the interaction between visible and mirror quarks, leptons and Higgs bosons, etc. (see [13, 18, 20, 24]). The search for mirror particles at LHC is discussed in Ref. [32].

Heavy Majorana neutrinos N_a are singlets of G_{SM} and G'_{SM} , and they can be messengers between visible and hidden worlds [13, 25].

The dynamics of the two worlds of our Universe, visible and hidden, is governed by the following action:

$$I = \int [L_{grav} + L'_{grav} + L_M + L'_M + L_{mix}], \quad (48)$$

where L_{grav} is the gravitational (left-handed) Lagrangian in the visible world, and L'_{grav} is the gravitational right-handed Lagrangian in the hidden world, L_M (L'_M) is the matter Lagrangian in the O(M)-world, and L_{mix} is the Lagrangian describing all mixing terms (see [18, 20]). Mixing terms give very small contributions to physical processes.

IV. TORSION IN PLEBANSKI'S FORMULATION OF GRAVITY

The gravitational theory with torsion can be presented in the Plebanski formalism by the following integral:

$$I_S = 2 \int \left(2S^I \wedge S^I + \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ} \right). \quad (49)$$

Using the partial integration and putting

$$\int \partial_\mu (T_{\nu\kappa\lambda}) dx^\mu \wedge dx^\nu \wedge dx^\kappa \wedge dx^\lambda = 0, \quad (50)$$

it is not difficult to show that

$$\int B^{IJ} \wedge F^{IJ} = 2 \int S^I \wedge S^I. \quad (51)$$

According to Eqs. (15) and (49), we have:

$$I_{dual\ GR} = 2 \int \left(B^{IJ} \wedge F^{IJ} + \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ} \right) = 2 \int \left(2S^I \wedge S^I + \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ} \right) = I_S. \quad (52)$$

This means that in the self-dual Plebanski formulation of the gravitational theory the following sectors of gravity coincide:

$$I_{GR} = I_{dual\ GR} = I_S. \quad (53)$$

Now it is obvious that we can exclude torsion as a separate dynamical variable. Here we see a close analogy of the geometry of the curved Riemann–Cartan space-time with torsion [8], with a structure of defects in a crystal [33–35].

A crystal can have two different types of topological defects [33]. A first type of such defects are translational defects called *dislocations*: a part of a single-atom layer is removed from the crystal and the remaining atoms relax to equilibrium under the elastic forces. A second type of defects is of the rotation type and called *disclinations*. They arise by removing an entire wedge from the crystal and re-gluing the free surfaces.

The geometry of the 4-dimensional Riemann–Cartan space-time is described by the direct generalizations of the translational and rotational defect gauge fields of the 3-dimensional crystal to the tetrads θ_μ^I and connections A_μ^{IJ} , which play the role of the translational and rotational defect gauge fields in the 4-dimensional Riemann–Cartan space-time [33].

The field strength of A_μ^{IJ} is given by the tensor (6), which describes the space-time curvature, and the field strength of θ_μ^I is the torsion given by Eq. (12). Torsion is presented by dislocations, and curvature by disclinations. But these defects are not independent of each other: a dislocation is equivalent to a disclination-antidisclination pair, and a disclination presents a string of dislocations. This explains why Einstein’s theory of gravity described only by curvature can be rewritten as Einstein’s ”teleparallel” theory of gravity [36] described only by torsion.

In summary, we wish to emphasize that if the Einstein–Hilbert Lagrangian is expressed in terms of the translational and rotational gauge fields, the tetrads θ_μ^I and the connection A_μ^{IJ} , then the Cartan curvature can be converted to torsion and back, totally or partially, by a new type of gauge transformation in Riemann–Cartan space-time [33].

In this general formulation, Einstein’s original theory is obtained by going to the zero-torsion gauge, while the ”teleparallel” theory is obtained in the gauge in which the Cartan curvature tensor vanishes. But any intermediate choice of the field A_μ^{IJ} is also allowed.

V. EQUATIONS OF MOTION

The equations of motion resulting from Plebanski’s action of gravity given by Eq. (28) (with $I \equiv I_{(left\ GR)}$) are:

$$\frac{\delta I}{\delta A^i} = D\Sigma^i = d\Sigma^i + \epsilon^{ijk} A^j \wedge \Sigma^k = 0, \quad (54)$$

$$\frac{\delta I}{\delta \psi_{ij}} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0, \quad (55)$$

$$\frac{\delta I}{\delta \Sigma^i} = F^i - (\Psi_{ij})^{-1} \Sigma^j = 0. \quad (56)$$

Eq. (54) states that A^i is the self-dual part of the spin connection compatible with the 2-forms Σ^i , where D is the exterior covariant derivative with respect to A^i . Eq. (55) implies that the 2-forms Σ^i can be constructed from tetrad one-forms giving (24), which fixes the conformal class of the space-time metric $g_{\mu\nu} = \eta_{IJ} \theta_\mu^I \otimes \theta_\nu^J$ defined by tetrads. Eq. (56) states that the trace-free part of the Ricci tensor vanishes [1–3].

The 2-form fields Σ^i can therefore be integrated out of Eq. (28). Thus, we are led to Plebanski’s gravity given by the form:

$$I_{(left\ GR)}(A, \psi) = \int \Psi_{ij} F^i \wedge F^j = \int (\Lambda \delta_{ij} + \psi_{ij})^{-1} F^i \wedge F^j, \quad (57)$$

discussed in Refs. [1–7], and

$$I_{(right\ GR)}(\bar{A}, \psi') = \int \Psi'_{ij} \bar{F}^i \wedge \bar{F}^j = \int (\Lambda' \delta_{ij} + \psi'_{ij})^{-1} \bar{F}^i \wedge \bar{F}^j. \quad (58)$$

A. Einstein's equations

Let us consider Einstein's equations in terms of the self-dual Plebanski's theory of gravity (we assume O-world). From Eq. (56) we obtain:

$$F^i \wedge \Sigma^i = (\Lambda \delta_{ij} + \psi_{ij}) \Sigma^i \wedge \Sigma^j. \quad (59)$$

Here,

$$iF^i \wedge \Sigma^i = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^i \Sigma_{\rho\sigma}^i \sqrt{-g} d^4x. \quad (60)$$

According to Ref. [3]:

$$i\Sigma^i \wedge \Sigma^j = 2\delta^{ij} \sqrt{-g} d^4x. \quad (61)$$

Taking into account that $\text{Tr} \psi_{ij} = 0$, we have:

$$i(\Lambda \delta_{ij} + \psi_{ij}) \Sigma^i \wedge \Sigma^j = 2(3\Lambda + \text{Tr} \psi_{ij}) \sqrt{-g} d^4x = 6\Lambda \sqrt{-g} d^4x, \quad (62)$$

what gives:

$$\frac{1}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^i \Sigma_{\rho\sigma}^i = 6\Lambda = \Lambda_0. \quad (63)$$

The curvature is a 2-form, and can be split in the basis of self-dual Σ^i and anti-self-dual $\bar{\Sigma}^i$:

$$F^i = X^{ij} \Sigma^j + \bar{X}^{ij} \bar{\Sigma}^j, \quad (64)$$

where:

$$X^{ij} = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^i \Sigma_{\rho\sigma}^j. \quad (65)$$

From Eq. (63)) we obtain:

$$\text{Tr} X^{ij} = \sum_i X^{ii} = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^i \Sigma_{\rho\sigma}^i. \quad (66)$$

Calculating the matrices X^{ij} , we obtain ten equations in Plebanski's variables, equivalent to the vacuum Einstein's equations:

$$\text{Tr} X^{ij} = \Lambda_0 \quad \text{and} \quad \bar{X}^{ij} = 0. \quad (67)$$

We have the following ten equations with matter fields, equivalent to Einstein's equations (40) :

$$\text{Tr} X^{ij} = \Lambda_0 - 2\pi GT \quad \text{and} \quad \bar{X}^{ij} = 0. \quad (68)$$

Here T is the trace of the stress-energy tensor of matter $T_{\mu\nu}$.

However, if we have two worlds OW and MW (or HW), then we have two Einstein's equations:

$$\text{Tr} X^{ij} = \Lambda_0^{(O)} - 2\pi GT^{(O)}, \quad (69)$$

and

$$\text{Tr} \bar{X}^{ij} = \Lambda_0^{(M)} - 2\pi GT^{(M)}, \quad (70)$$

where

$$\text{Tr} \bar{X}^{ij} = \sum_i \bar{X}^{ii} = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \bar{F}_{\mu\nu}^i \bar{\Sigma}_{\rho\sigma}^i. \quad (71)$$

B. Newtonian gravity

In the non-relativistic limit we have $A_0^i = \partial_i \Phi(x)$, where $\Phi(x)$ is given by $g_{00} = 1 + 2\Phi(x)$. Then

$$\Delta \Phi(x) = 4\pi G \rho, \quad (72)$$

what leads to the Newtonian gravitational potential of the particle with mass M and $G_N = G$:

$$V(r) \equiv \Phi(r) \approx -G_N \frac{M}{r}. \quad (73)$$

This result comes from Eq. (68).

VI. DIFFEOMORPHISM INVARIANT GAUGE THEORY OF GRAVITY

Gravity is not a gauge theory of the usual type. The carriers of force in a usual gauge theory are spin one particles. Moreover, in the electromagnetic field theory, for example, there are two types of charged objects, negatively and positively charged particles, which interact by exchange of carriers of force. As a result, particles can either repel, or attract. In contrast, there is only one type of charge in gravity, and we have only attraction.

However, scattering amplitudes for gravitons can be expressed as squares of amplitudes for gluons (see for example [37, 38]): the closed string theory describes gravity, and the open string theory is a gauge theory. The relationship is not direct, but it exists, and it is not easy to find a Lagrangian version of the correspondence. In the Plebanski–Ashtekar formalism the gravity/gauge theory relationship was developed in Refs. [39, 40].

Finally, it was realized that Plebanski’s formulation of GR [1] can be integrated out to obtain a ‘pure connection’ formulation of GR, where the only dynamical field is an $SU(2)$ connection [40, 41]. The result is a completely new perspective on general relativity, in which GR was reformulated as a *diffeomorphism invariant gauge theory*.

The pure connection formulation of GR [40] was further developed. It was shown in Ref. [42] that in this case we have not a single diffeomorphism invariant gauge theory, but an infinite parameter class of them. All these theories have the same number of propagating degrees of freedom (DOF). For any theory of this type the phase space is that of an $SU(2)$ gauge theory. However, in addition to the usual $SU(2)$ gauge rotations, there are also diffeomorphisms acting on the phase space variables, which reduce the number of propagating DOF from 6 of the $SU(2)$ gauge theory to 2 of GR [43].

In the present paper we try to develop the gravity/gauge theory correspondence.

Using the imaginary time $x_0 = it$, e.g. assuming the Euclideanized self-dual Plebansky action (57), we can calculate the partition function:

$$Z = \int [\mathcal{D}A][\mathcal{D}\psi] e^{-S} = \int [\mathcal{D}A][\mathcal{D}\psi] \exp \left[- \int (\Lambda \delta_{ij} + \psi_{ij})^{-1} F^i \wedge F^j \right]. \quad (74)$$

Here,

$$F^i \wedge F^j = \frac{1}{4} F_{\mu\nu}^i F_{\rho\sigma}^j dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma = \frac{1}{4} F_{\mu\nu}^i F_{\rho\sigma}^j \epsilon^{\mu\nu\rho\sigma} \sqrt{g} d^4x. \quad (75)$$

The Hodge-star operation in the curved space-time determines the following dual tensor:

$$F^{*j\mu\nu} = \frac{1}{2} \sqrt{g} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^j, \quad (76)$$

and we have:

$$F^i \wedge F^j = \frac{1}{2} F_{\mu\nu}^i F^{*j\mu\nu} d^4x \equiv \frac{1}{2} F^i \cdot F^{*j} d^4x. \quad (77)$$

The requirement of self-duality in the curved space-time is absent in gravity:

$$F^{*j\mu\nu} \neq F^{j\mu\nu}.$$

Then we obtain the following partition function:

$$Z = \int [\mathcal{D}A][\mathcal{D}\psi] e^{-I} \approx \int [\mathcal{D}A][\mathcal{D}\psi] \exp \left[- \frac{1}{2} \int (\Lambda \delta_{ij} + \psi_{ij})^{-1} F^i \cdot F^{*j} d^4x \right]. \quad (78)$$

In the limit of large $F^2 \simeq 1$ ($M_{Pl}^{red} = 1$ in our units), the effective charge g_{eff} of gravitational fields is asymptotically small and equal to:

$$g_{eff}^2 \approx \frac{\Lambda}{2} \ll 1, \quad (79)$$

which follows from Eq. (78): minimal g_{eff} corresponds to $\psi_{ij} = 0$. In such a regime (at small distances $r \sim \lambda_{Pl}$) the space-time is Euclidean. This means that in our (visible) Universe, we have respectively the flat Minkowski space-time, and we can consider the condition of self-duality: $F = F^*$. Then the asymptotic gravitational Lagrangian is:

$$L_{eff}^{as} = -\frac{1}{4g_{eff}^2} F_{\mu\nu}^i F_{\mu\nu}^i \approx -\frac{1}{2\Lambda} F_{\mu\nu}^i F_{\mu\nu}^i. \quad (80)$$

However, at large distances the theory of gravity is more complicated and the effective Lagrangian depends on $F^i \cdot F^{*i}$ (see for example Refs. [41]).

A. Coupling to matter

A complete theory of gravity can be constructed only if all matter fields are incorporated into the theory. Then it is necessary to construct the Lagrangian of the Universe considered in Eq. (48).

In the Plebanski–Ashtekar formulation, the fundamental objects are a rule for parallel transport with a connection in the curved space. An operator which compares fields at different points is an operator of the parallel transport between the points x and y :

$$U(x, y) = P e^{i \int_{C_{xy}} \hat{A}_\mu(\tilde{x}) d\tilde{x}^\mu}, \quad (81)$$

where P is the path ordering operator, C_{xy} is a curve from point x till point y and

$$\hat{A}_\mu(x) = A_\mu^i \frac{\sigma^i}{2},$$

with σ^i being the Pauli matrices.

The operator:

$$W = P e^{i \oint \hat{A}_\mu(x) dx^\mu} \quad (82)$$

is the well-known Wilson loop. The graviton is related to a closed string.

In the case of a spinor field $\chi(x)$ interacting with the gauge field A_μ^i , we have an additional gauge invariant observable:

$$\bar{\chi}(y) P e^{i \int_{C_{xy}} \hat{A}_\mu(\tilde{x}) d\tilde{x}^\mu} \chi(x). \quad (83)$$

In the theory with two worlds, ordinary and mirror (or hidden), the self-dual gravity with connection A_μ^i interacts with the left-handed spinors χ_L and χ'_L of the visible and mirror worlds, respectively, while the anti-self-dual gravity with connection \bar{A}_μ^i interacts with the right-handed spinors χ_R and χ'_R of the O- and M-worlds, respectively.

Due to CP violation, the following cross-sections with ordinary quarks q and mirror quarks q' :

$$\sigma(q + q \rightarrow q' + q') \neq \sigma(\bar{q} + \bar{q} \rightarrow \bar{q}' + \bar{q}') \quad (84)$$

are different from each other, what is essential for the baryogenesis.

B. Quantum gravity and renormalization problem

In the framework of quantum field theory, and using the standard techniques of perturbative calculations, one finds that gravitation is non-renormalizable.

The theory of Loop Quantum Gravity (LQG) is a way of quantizing the Plebanski–Ashtekar gravity. In LQG, space is represented by a spin network, evolving over time in discrete steps [44, 45]. The phase space version [42] of

the new 'pure connection' viewpoint on GR in the Plebanski formalism has led to the approach of LQG [45]. This class of theories is closed under the renormalization [41].

In Refs. [44] and [45] it was argued that it is possible to use Wilson loops as the basis for a nonperturbative quantization of gravity. Explicit (spatial) diffeomorphism invariance of the vacuum state plays an essential role in the regularization of the Wilson loop states. An explicit basis of states of quantum geometry was obtained, and the geometry was shown to be quantized – that is, the (non-gauge-invariant) quantum operators representing area and volume have a discrete spectrum. In this context, spin networks arose as a generalization of Wilson loops. Plebanski's formalism is a starting point for "spinfoam" models (see [45] and references therein).

Should LQG succeed as a quantum theory of gravity, the known matter fields will have to be incorporated into the theory.

Considering the problem of renormalizability of quantum gravity, one can construct a model of multi-gravitons (see for example [46, 47]) with N massive gravitons.

VII. SUMMARY AND CONCLUSIONS

In this paper we have explained the main idea of Plebanski [1] to construct the 4-dimensional theory of gravity described by the gravitational action with an integrand presented by a product of two 2-forms, which are constructed from the tetrads θ^I and the connection A^{IJ} considered as independent dynamical variables. Both A^{IJ} and θ^I are 1-forms. The tetrads θ_μ^I were used instead of the metric $g_{\mu\nu}$. We considered the Minkowski space with the group of symmetry $SO(1, 3)$.

We have reviewed the well-known Plebanski BF-theory of general relativity (GR) and constructed the gravitational actions of the different theories of pure gravity: ordinary, dual and 'mirror' ones, as well as the gravity with torsion. We have considered the self-dual left-handed gravity of the Ordinary World (OW) and the anti-self-dual right-handed gravity of the Mirror World (MW) with broken mirror parity. We have shown that in the Plebanski self-dual formulation of gravity the ordinary and dual gravitational actions coincide.

We reviewed the close analogy of geometry of space-time in GR with a structure of defects in a crystal [33]. We have considered the translational defects – dislocations, and the rotational defects – disclinations, in the 4-dimensional crystals. The crystalline defects represent a special version of the curved space-time – the Riemann–Cartan space-time with torsion [8]. The world crystal is a model for Einstein's gravitation which has a new type of gauge symmetry with zero torsion as a special gauge, while a zero connection (with zero Cartan curvature) is another equivalent gauge with nonzero torsion which corresponds to the Einstein's theory of "teleparallelism" [36]. Here we showed that in the Plebanski formulation, the phase of gravity with torsion is equivalent to the ordinary or dual gravity, and we can exclude torsion as a separate dynamical variable.

We have considered the equations of motion which follow from the Plebanski action of gravity with the tetrads, self-dual connection and auxiliary fields ψ_{ij} . The vacuum Einstein's equations were obtained in the framework of the Plebanski theory of gravity. Integrating out the tetrads we constructed the gravitational action containing only the connection and the auxiliary fields. The integration of the action over the auxiliary fields ψ_{ij} leads to a new type of formulation of the gravitational theory with a 'pure connection'. Here the diffeomorphism invariant gauge theory of gravity is developed where the only dynamical field is an $SU(2)$ spin connection [40, 41]. This theory is a completely new perspective on GR.

We have calculated the partition function and the effective Lagrangian of this 4-dimensional gravity. We have considered the asymptotic limit of this theory: the large values $F^2 \sim M_{Pl}^4$, which correspond to small transPlanckian distances $r \sim \lambda_{Pl}$, where λ_{Pl} is the Planck length. At these small distances, the connection fields A_μ^i exist in the flat (Euclidean or Minkowski) space-time, and the effective gravitational coupling constant is given by the cosmological constant Λ : $g_{eff} = \Lambda/2$. At large distances we envisage a more complicated theory of gravity. A complete theory of gravity has to be constructed only with couplings to matter.

Finally, we recalled the role of Plebanski's formalism in the theory of Loop Quantum Gravity, which is a way of quantizing the Plebanski–Ashtekar theory of gravity.

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